Laurea Magistrale in Scienze Statistiche a.a. 2022/23
and
PhD Course in Statistical Sciences – cycle XXXVIII, 2023

Theory and Methods of Inference

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Syllabus

Some prerequisites. Empirical distribution function. Convergence of sums of r.v.'s. Order statistics. Density functions. Scale and location families. Exponential families. Multivariate normal distributions. Parametric inference: basics.

Statistical models and uncertainty in inference. Paradigms of inference: the Bayesian and frequentist paradigms. Model specification in Bayesian and frequentist inference. Frequentist evaluation of uncertainty and distribution problems. Simulation. Asymptotic approximations and delta method.

Generating functions, moment approximations, transformations. Moments, cumulants and generating functions. Edgeworth and Cornish-Fisher expansions. Notations $O_p()$ and $o_p()$. Approximations of moments and transformations. Laplace approximation.

Data and model reduction. Dominated statistical models and density factorizations. Sufficiency. Distribution constant statistics. Completeness. Data and model reduction with nuisance parameters.

Exponential and group families. Exponential families and exponential tilting. Mean value mapping and variance function. Multiparameter exponential families. Sufficiency and completeness in exponential families. Marginal and conditional distributions. Exponential dispersion families. Generalized linear models. Groups of transformations. Orbits and maximal invariants. Simple group families. Composite group families.

Likelihood: observed and expected quantities, exact properties. Observed likelihood quantities in exponential and group families. Examples with various models and observation schemes: grouped data, censored data, sequential sampling, stochastic processes. Invariance properties. Likelihood and sufficiency. Expected quantities and exact sampling properties. Universal bounds. Orthogonal parameters and mixed parameterization in exponential families. Reparameterizations. Ancillary statistics. Conditional inference in scale and location families.

Likelihood inference: asymptotics. Likelihood inference procedures. Consistency of the maximum likelihood estimator. First-order asymptotics and related inference procedures. Profile likelihood. Asymptotically equivalent forms and one-sided versions. First-order asymptotic theory in exponential families. Non-regular models. Approximate conditional inference and higher-order asymptotics.

Bayesian Inference. Asymptotic approximations. Non-informative priors. Empirical Bayes methods. Inference based on the posterior distribution: point estimation and credibility regions, hypothesis testing and the Bayes factor. Prediction. Linear models.

Likelihood and Bayesian inference in R. Scalar parameter examples: log likelihood, plot of the log likelihood, MLE and observed/expected information, Wald confidence intervals, deviance confidence regions, simulation, numerical optimization methods, significance function. Vector parameters: plot of the log likelihood, parameter estimates, simulation. Parameter of interest and profile likelihood. Simulation. Stratified models. Likelihood and parametric bootstrap. EM algorithm with applications to censored data and mixture models. Bayesian inference: posterior summaries, simulation from the posterior (rejection sampling).

Estimating equations and pseudo-likelihoods. Misspecification. Estimating equations. Quasi likelihood. Pseudo-likelihoods. Composite likelihood. Empirical likelihood. Conditional and marginal likelihood in exponential and group families. Profile and integrated likelihoods.

Decision paradigms.. Statistical decision problems and Bayes decision rules. Efficient estimators: Cramér-Rao lower bound, asymptotic efficiency, Godambe efficiency, Rao-Blackwell-Lehmann-Scheffé theorem, other constraints for point estimation. Optimal tests: Neyman-Pearson lemma, composite hypotheses: families with monotone likelihood ratio, locally most powerful tests, two-sided alternatives, other constraint criteria. Optimal confidence regions. Procedures with finite sample optimality properties in exponential and group families.

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