In survival and event-history analysis, the occurrence of one or several events is studied, and the occurrence is described by hazard rates. Standard models consider hazards that vary along one main time scale, usually chosen based on previous knowledge of the process under study. However, in many applications multiple time scales are relevant for the occurrence of an event. For example, marital fertility is influenced by the age of the mother and the time since marriage, or mortality from chronic diseases varies with the age of the patient and the duration of illness. Traditional models select one of the time scales as dominant and incorporate other time scales as a time-varying covariate that changes at some discrete time points, commonly in a proportional hazards setting. Alternatively, time-varying effects of a fixed time at entry in the risk set are included.

A different approach treats all time scales on an equal footing and, in the case of two time scales, models the hazard as a bivariate function over both scales. The two-dimensional hazard is assumed to vary smoothly over the time axes, but otherwise the interplay between the time scales is unrestricted. The new model thus avoids the need to single out one of the time scales as dominant and provides a tool to study even complex interactions between the time domains.

We employ penalized splines (P-splines) in order to estimate the smooth bivariate hazard. The approach allows to incorporate standard observations schemes, such as right-censoring and left-truncation. Further, the model is extended to hazard regression where a two-dimensional baseline hazard is modulated by covariates in a proportional hazards setting. It can be applied to the analysis of a single event, but was also extended to competing risks.

As an example, I will present a model for transitions from cohabitation to marriage or separation simultaneously over the age of the individual and the duration of the cohabitation, using data from the German Family Panel.