PhD School in Statistics
cycle XXIX, 2014

Theory and Methods of Statistical Inference

Instructors: B. Liseo, L. Pace, A. Salvan (course coordinator), N. Sartori, A. Tancredi, L. Ventura

Syllabus

Some prerequisites: Empirical distribution function. Convergence of sums of r.v.s. Order statistics. Density function and Radon-Nikodym theorem. Scale and location families. Exponential families. Multivariate normal distributions. Parametric inference: basics.

Statistical models: data variability and uncertainty in inference: Statistical models. Paradigms of inference. The Fisherian paradigm. Model specification (data variability). Levels of specification. Problems of distribution (statistical variability). Simulation. Asymptotic approximations and delta method.

Generating functions, moment approximations, transformations: Moments, cumulants and their generating functions. Generating functions of sums of independent random variables. Edgeworth and Cornish-Fisher expansions. Notations $O_p(\cdot)$ and $o_p(\cdot)$. Approximations of moments and transformations.

Likelihood: observed and expected quantities, exact properties: Dominated statistical models. Sufficiency. Likelihood: observed quantities. Examples: a two-parameter model, grouped data, censored data, sequential sampling, Markov chains, Poisson processes. Likelihood and sufficiency. Invariance properties. Expected likelihood quantities and exact sampling properties. Reparameterizations.

Likelihood inference (first-order asymptotics): Likelihood inference procedures. Consistency of the maximum likelihood estimator. Asymptotic distribution of the maximum likelihood estimator. Asymptotic distribution of the log-likelihood ratio: simple null hypothesis, likelihood confidence regions, comparisons among asymptotically equivalent forms, non-null asymptotic distributions, composite null hypothesis (nuisance parameters), profile likelihood, asymptotically equivalent forms and one-sided versions, testing constraints on the components of the parameter. Non-regular models.

Likelihood: numerical and graphical aspects in R Scalar and vector parameters: A scalar parameter example: log likelihood, plot of the log likelihood, MLE and observed/expected information, Wald confidence intervals, deviance confidence regions, simulation, numerical optimization methods, significance function. A vector parameter example: plot of the log likelihood, parameter estimates, simulation.

Likelihood: numerical and graphical issues in R - Parameter of interest and profile likelihood: Parameter of interest and profile likelihood. Examples in the Weibull model. Deviance intervals: simulation. Stratified models.

Estimating equations and pseudolikelihoods: Misspecification. Estimating equations. Quasi likelihood. Pairwise likelihood. Empirical likelihood.

Data and model reduction by marginalization and conditioning: Distribution constant statistics. Completeness. Ancillary statistics. Data and model reduction with nuisance parameters: lack of information with nuisance parameters, pseudo-likelihoods. Marginal likelihood. Conditional likelihood.

The frequency-decision paradigm: Statistical decision problems. Efficient estimators: Cramér-Rao lower bound, asymptotic efficiency, Godambe efficiency, Rao-Blackwell-Lehmann-Scheffé theorem, other constraints for point estimation. Optimal tests: Neyman-Pearson lemma, composite hypotheses: families with monotone likelihood ratio, locally most powerful tests, two-sided alternatives, other constraint criteria. Optimal confidence regions.

Exponential families, Exponential dispersion families, Generalized linear models: Exponential families of order 1. Mean value mapping and variance function. Multiparameter exponential families. Marginal and conditional distributions. Sufficiency and completeness. Likelihood and exponential families: likelihood quantities, conditional likelihood, profile likelihood and mixed parameterization. Procedures with finite sample optimality properties. First-order asymptotic theory. Exponential dispersion families. Generalized linear models.

Group families: Groups of transformations. Orbits and maximal invariants. Simple group families and conditional inference. Composite group families and marginal inference.

Higher order asymptotics: Laplace expansions. Approximation of marginal likelihood. Bayesian interpretation of approximate conditional likelihood. Saddlepoint expansion for density functions. Improvements over classical asymptotic theory. The p^* formula. Tail probabilities and modified directed likelihood. Bartlett adjustment. Modified profile likelihood.

Bayesian Inference: Statistical models and prior information. Inference based on the posterior distribution. Choice of the prior distribution. Point and interval estimation. Hypothesis testing and the Bayes factor. Linear models. Model selection. Monte Carlo. MCMC. Generalized linear models. Hierarchical models and excheangeability.

References

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Young, G.A. and Smith, R.L. (2005). *Essentials of Statistical Inference.* Cambridge University Press, Cambridge.

References for Bayesian Inference

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Lectures and topics

date		topic	instructor
13/03/14	14.30–17.00	course presentation, some prerequisites	AS
14/03/14	10.00–11.00	*Sweave*: mixing R and LaTeX	NS
17/03/14	10.30–13.00	statistical models: data variability and sampling variability	AS
18/03/14	10.30–13.00	generating functions, approximation of moments, transformations	AS
20/03/14	10.30–13.00	likelihood: observed and expected quantities, exact sampling properties	AS
26/03/14	10.30–13.00	likelihood: observed and expected quantities, exact sampling properties	AS
28/03/14	09.30–12.00	likelihood inference (first-order asymptotics)	NS
01/04/14	09.30–12.00	likelihood inference (first-order asymptotics)	NS
02/04/14	09.30–12.00	lab. on "likelihood: graphical and numerical techniques with R, I"	NS
08/04/14	09.30–12.00	estimating equations and pseudolikelihoods	NS
09/04/14	09.30–12.00	lab. on "likelihood: graphical and numerical techniques with R, II"	NS
15/04/14	10.30–13.00	data and model reduction by marginalization and conditioning	AS
17/04/14	10.30–13.00	data and model reduction by marginalization and conditioning	AS
29/04/14	09.30–12.30	Bayesian inference. Introduction: prior and posterior distributions	LV
06/05/14	09.30–12.30	choice of priors; noninformative priors	LV
08/05/14	09.30–12.30	linear models; model selection	LV
09/05/14	10.30–13.00	the frequency decision paradigm	AS
13/05/14	10.30–13.00	the frequency decision paradigm	AS
14/05/14	14.30–17.30	exchangeability and hierarchical models	AT
15/05/14	10.00–12.30	Monte Carlo Integration for Bayesian inference	AT
15/05/14	14.30–17.00	lab session	AT
20/05/14	10.30–13.00	the frequency decision paradigm	AS
21/05/14	14.30–17.30	MCMC methods, Metropolis-Hastings Algorithms	AT
22/05/14	10.00–12.30	Gibbs sampler	AT
22/05/14	14.30–17.00	lab session	AT
27/05/14	10.30–13.00	exponential families	AS
29/05/14	10.30–13.00	exponential families	AS
03/06/14	10.30–13.00	exponential dispersion models, generalized linear models	AS
05/06/14	10.30–13.00	group families	AS
06/06/14	10.30–13.00	group families	AS
(Autumn)		higher-order likelihood theory	NS
24/06/14	09.30–12.30	written exam	
25/06/14	09.30–12.30	practical exam	
26/06/14	10.00–12.00	Writing papers and reports + paper assignment	AS+NS
??/09/14	00.00–00.00	oral presentations	

Instructors: AS is Alessandra Salvan, NS is Nicola Sartori, LV is Laura Ventura, AT is Andrea Tancredi.

Luigi Pace (LP) collaborates in lecture material, in the final exam and in paper assignment and evaluation. Brunero Liseo (BL) collaborates in lecture material for the Bayesian part.